Effect of Higher-Order Corrections on the Propagation of Nonlinear Dust-Acoustic Solitary Waves in Mesospheric Dusty Plasmas

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Z. Naturforsch. **61a,** 316 – 322 (2006); received March 6, 2006

The contribution of the higher-order correction to nonlinear dust-acoustic waves are studied using the reductive perturbation method in an unmagnetized collisionless mesospheric dusty plasma. A Korteweg – de Vries (KdV) equation that contains the lowest-order nonlinearity and dispersion is derived from the lowest order of perturbation, and a linear inhomogeneous (KdV-type) equation that accounts for the higher-order nonlinearity and dispersion is obtained. A stationary solution is achived via renormalization method.

Key words: Mesospheric Dusty Plasma; Dust-Acoustic Waves; Renormalization Method; Higher-Order Correction; Solitary Solution.

1. Introduction

Studies of numerous collective processes [1, 2] in dust contaminated plasmas have been of significant interest in connection with linear and nonlinear waves that are observed in laboratory, astrophysical and space environments; see for instance [3 – 12]. These dust grains are positively and negatively charged because of a number of charging processes, such as photoelectric emission stimulated by ultraviolet radiation, thermionic emission induced by radiative heating, secondary emission of electrons from the surface of the dusty grains and collisional charging by electrons and ions [13 – 15]. It has been shown both theoretically and experimentally that the presence of charged dust grains does not only modify the existing plasma wave spectra [16], but introduces also new eigenmodes, such as dust-aoustic (DA) waves [17, 18], dust-ion-acoustic (DIA) waves [19, 20], dust-lattice (DL) waves [21, 22].

Recently, Mamun and Shukla [22] investigated theoretically the properties of linear and nonlinear arbitrary amplitude dust-acoustic waves that propagate in a mesospheric dusty plasma. In the present work, taking into account the same mesospheric dusty plasma system suggested in [23], we make use of the reductive perturbation theory for a small but finite amplitude. We will derive the Korteweg – de Vries (KdV) equation that contains the lowest-order nonlinearity and dispersion. As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KdV equation. To overcome this deviation, the higher-order nonlinearity and dispersion must be taken into account. A stationary solution for the resulting equations has been achieved via the renormalization method [24 – 30].

The present paper proceeds as follows: in Section 2, the basic set of equations is introduced and the KdV equation is derived. In Section 3 we continue to a higher-order reductive perturbation theory to obtain an equation for the second-order potential. In Section 4 we apply the renormalization method to obtain a stationary solution. Finally, some conclusions and remarks are given in Section 5.

2. Basic Equations and KdV Equation

Let us consider an unmagnetized collisionless dusty plasma consisting of a two-component, uniform, mesospheric dusty plasma containing positively and negatively charged dust fluids. We assume that the negatively charged dust particles are much more massive than the positively charged ones [7, 11]. The basic equations are:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1a}$$

$$\mu\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+\frac{\partial \phi}{\partial x}+\frac{1}{n}\frac{\partial p}{\partial x}=0, \tag{1b}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+3p\frac{\partial u}{\partial x}=0, \tag{1c}$$

for the positively charged dust plasma and

$$\frac{\partial N}{\partial t}+\frac{\partial (NV)}{\partial x}=0, \tag{2a}$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}-\frac{\partial \phi}{\partial x}=0. \tag{2b}$$

for the negatively charged dust plasma.

Equations (1) and (2) are supplemented by Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2}=N-n. \tag{3}$$

In the above equations n, N, u, V, ϕ, and p are the number densities of positive and negative dust grains, the velocities of positive and negative dust grains, the electric potential and the thermal pressure of the positive dust fluid. Here n and N are normalized by equilibrium values n_0 and N_0, u and V are normalized by $C_s=\sqrt{\mu}V_T$ [$\mu=Z_2m_1/Z_1m_2, V_T=(k_BT/m_1)^{\frac{1}{2}}$, Z_1 (Z_2) represents the number of the positive (negative) charges onto the positive (negative) dust grain surface, m_1 (m_2) represents the mass of the positive (negative) dust particle, k_B is the Boltzmann constant, T is the temperature of the positive dust fluid], p is normalized by $n_{10}k_BT$, ϕ is normalized by k_BT/Z_1e, x is the space variable normalized by $\lambda_{ID}=(k_BT_I/4pn_{10}Z_1)^{\frac{1}{2}}$, t is the time variable normalized by $\omega_{p2}^{-1}=(m_2/4\pi n_{20}Z_2^2e^2)^{\frac{1}{2}}$.

To derive the KdV equation describing the behavior of the system for longer times and small but finite amplitude DA waves, we introduce the slow stretched co-ordinates

$$\tau=\varepsilon^{\frac{3}{2}}t, \quad \xi=\varepsilon^{\frac{1}{2}}(x-\lambda t), \tag{4}$$

where ε is a small dimensionless expansion parameter and λ is the speed of DA waves. All physical quantities appearing in (1)–(3) are expanded as power series in ε about their equilibrium values as:

$$n=1+\varepsilon n_1+\varepsilon^2 n_2++\varepsilon^3 n_3\ldots,$$
$$u=\varepsilon u_1+\varepsilon^2 u_2+\varepsilon^3 u_3+\ldots,$$
$$N=1+\varepsilon N_1+\varepsilon^2 N_2+\varepsilon^3 N_3+\ldots,$$
$$V=\varepsilon V_1+\varepsilon^2 V_2+\varepsilon^3 V_3+\ldots,$$
$$p=1+\varepsilon p_1+\varepsilon^2 p_2+\varepsilon^3 p_3+\ldots,$$
$$\phi=\varepsilon \phi_1+\varepsilon^2 \phi_2+\varepsilon^3 \phi_3+\ldots. \tag{5}$$

We impose the boundary conditions that as

$$|\xi|\to\infty, \quad n=N=1, \quad p=1, \quad u=V=0, \quad \phi=0. \tag{6}$$

Substituting (4) and (5) into (1)–(3), and equating coefficients of like powers of ε, then, from the lowest-order equations in ε, the following results are obtained:

$$n_1=\frac{-1}{\lambda^2}\phi_1, \quad u_1=\frac{-1}{\lambda}\phi_1, \quad N_1=\frac{-1}{\lambda^2}\phi_1, \quad V_1=\frac{-1}{\lambda}\phi_1, \quad p_1=\frac{-3}{\lambda^2}\phi_1. \tag{7}$$

Poisson's equation gives the linear dispersion relation

$$\lambda^2(1+\mu)=3. \tag{8}$$

If we consider the coefficients of $O(\varepsilon^2)$, we obtain with the aid of (7) the following set of equations:

$$-\lambda\frac{\partial n_2}{\partial \xi}+\frac{\partial u_2}{\partial \xi}-\frac{1}{\lambda^2}\frac{\partial \phi_1}{\partial \tau}+\frac{1}{\lambda^3}\phi_1\frac{\partial \phi_1}{\partial \xi}=0, \tag{9a}$$

$$\frac{-\mu}{\lambda}\frac{\partial \phi_1}{\partial \tau}+\frac{\partial p_2}{\partial \xi}-\lambda\mu\frac{\partial u_2}{\partial \xi}+\frac{\mu}{\lambda^2}\phi_1\frac{\partial \phi_1}{\partial \xi}+\frac{\partial \phi_2}{\partial \xi}=0, \tag{9b}$$

$$-\lambda\frac{\partial p_2}{\partial \xi}+\frac{12}{\lambda^3}\phi_1\frac{\partial \phi_1}{\partial \xi}+3\frac{\partial u_2}{\partial \xi}-\frac{3}{\lambda^2}\frac{\partial \phi_1}{\partial \tau}=0. \tag{9c}$$

$$-\frac{1}{\lambda^2}\frac{\partial \phi_1}{\partial \tau}-\lambda\frac{\partial N_2}{\partial \xi}+\frac{\partial V_2}{\partial \xi}+\frac{1}{\lambda^3}\phi_1\frac{\partial \phi_1}{\partial \xi}=0, \tag{10a}$$

$$-\frac{1}{\lambda}\frac{\partial \phi_1}{\partial \tau}-\lambda\frac{\partial V_2}{\partial \xi}+\frac{1}{\lambda^4}\phi_1\frac{\partial \phi_1}{\partial \xi}-\frac{\partial \phi_2}{\partial \xi}=0, \tag{10b}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2}+n_2-N_2=0. \tag{11}$$

Eliminating the second-order perturbed quantities n_2, u_2, N_2, V_2 and ϕ_2 in equations (9)–(11), we obtain the following KdV equation for the first-order perturbed potential:

$$\frac{\partial \phi_1}{\partial \tau}+\mathbf{A}\,\phi_1\frac{\partial \phi_1}{\partial \xi}+\frac{\mathbf{B}}{2}\frac{\partial^3 \phi_1}{\partial \xi^3}=0, \tag{12}$$

where

$$\mathbf{A}=\frac{-2}{\lambda}, \quad \mathbf{B}=\frac{\lambda^5}{3}. \tag{13}$$

3. Linear Inhomogeneous Equation

In this section we start by writing down the second-order perturbed quantities n_2, u_2, p_2, N_2 and V_2 in terms of ϕ_1 and ϕ_2. They are expressed as

$$n_2 = \frac{-3(-3+\mu\lambda^2)\phi_1^2 + 2\lambda^4(3\phi_2 + \lambda^4 \frac{\partial^2\phi_1}{\partial\xi^2})}{6\lambda^4(-3+\mu\lambda^2)}, \quad (14a)$$

$$u_2 = \frac{-3(-3+\mu\lambda^2)\phi_1^2 + 6\lambda^4\phi_2 + \lambda^6(3+\mu\lambda^2)\frac{\partial^2\phi_1}{\partial\xi^2}}{6\lambda^3(-3+\mu\lambda^2)}, \quad (14b)$$

$$p_2 = \frac{3\phi_1^2}{2\lambda^4} + \frac{3\phi_2 + \mu\lambda^4 \frac{\partial^2\phi_2}{\partial\xi^2}}{-3+\mu\lambda^2}, \quad (14c)$$

$$N_2 = -\frac{3\phi_1^2 + 6\lambda^2\phi_2 - 2\lambda^6(3+\mu\lambda^2)\frac{\partial^2\phi_1}{\partial\xi^2}}{6\lambda^4}, \quad (14d)$$

$$V_2 = -\frac{3\phi_1^2 - 6\lambda^2\phi_2 + \lambda^6 \frac{\partial^2\phi_1}{\partial\xi^2}}{6\lambda^3}. \quad (14e)$$

If we now go to the higher-order in ε, we obtain the following equations:

$$\frac{\partial n_2}{\partial\tau} + u_2\frac{\partial n_1}{\partial\xi} + u_1\frac{\partial n_2}{\partial\xi} - \lambda\frac{\partial n_3}{\partial\xi} + n_2\frac{\partial u_1}{\partial\xi} + n_1\frac{\partial u_2}{\partial\xi} + \frac{\partial u_3}{\partial\xi} = 0, \quad (15a)$$

$$\frac{\partial p_2}{\partial\tau} + u_2\frac{\partial p_1}{\partial\xi} + u_1\frac{\partial p_2}{\partial\xi} - \lambda\frac{\partial p_3}{\partial\xi} + 3p_2\frac{\partial u_1}{\partial\xi} + 3p_1\frac{\partial u_2}{\partial\xi} + 3\frac{\partial u_3}{\partial\xi} = 0, \quad (15b)$$

$$\frac{\mu\partial u_2}{\partial\tau} - \frac{n_1\partial p_2}{\partial\xi} + \frac{\partial p_3}{\partial\xi} + \frac{\mu u_2\partial u_1}{\partial\xi} + \frac{\mu u_1\partial u_2}{\partial\xi} - \frac{\mu\lambda\partial u_3}{\partial\xi} + \frac{\partial\phi_3}{\partial\xi} = -\frac{1}{2}\frac{n_1^2\partial p_1}{\partial\xi} + \frac{n_2\partial p_1}{\partial\xi}. \quad (15c)$$

$$\frac{\partial N_2}{\partial\tau} + V_2\frac{\partial N_1}{\partial\xi} + V_1\frac{\partial N_2}{\partial\xi} - \lambda\frac{\partial N_3}{\partial\xi} + N_2\frac{\partial V_1}{\partial\xi} + N_1\frac{\partial V_2}{\partial\xi} + \frac{\partial\phi_3}{\partial\xi} = 0, \quad (16a)$$

$$\frac{\partial V_2}{\partial\tau} + V_2\frac{\partial V_1}{\partial\xi} + v_1\frac{\partial V_2}{\partial\xi} - \lambda\frac{\partial V_3}{\partial\xi} - \frac{\partial\phi_3}{\partial\xi} = 0, \quad (16b)$$

$$\frac{\partial^2\phi_2}{\partial\xi^2} - N_3 + n_3 = 0. \quad (16c)$$

Eliminating n_3, u_3, N_3, V_3 and ϕ_3 from (15) and (16), we finally obtain, with the aid of (7), (9), (10) and (11), a linear inhomogeneous equation for the second-order perturbed potential ϕ_2:

$$\widetilde{L}(\phi_1)\phi_2 \equiv \frac{\partial\phi_2}{\partial\tau} + A\frac{\partial(\phi_1\phi_2)}{\partial\xi} + \frac{B}{2}\frac{\partial^3\phi_2}{\partial\xi^3} - S(\phi_1) = 0, \quad (17)$$

where

$$S(\phi_1) = A_1\frac{\partial\phi_1^3}{\partial\xi} + A_2(\frac{\partial}{\partial\xi}\phi_1\frac{\partial^2\phi_1}{\partial\xi^2}) + A_3\phi_1(\frac{\partial^3\phi_1}{\partial\xi^3}) - A_4\frac{\partial^5\phi_1}{\partial\xi^5}. \quad (18)$$

The coefficients A_i (where $i = 1,2,\ldots,4$) are given by

$$A_1 = \frac{-5}{12\lambda^3}, \quad A_2 = \frac{\lambda^3\mu}{6}, \quad A_3 = \frac{-\lambda^3}{3}, \quad A_4 = -\left(\frac{\lambda^{11}}{72} - \frac{\mu\lambda^9}{24} + \frac{\mu\lambda^{11}}{108} - \frac{\mu^2\lambda^{11}}{216}\right). \quad (19)$$

Thus, we have reduced the basic set of fluid equations (1)–(3) to a nonlinear KdV equation (12) for ϕ_1 and a linear inhomogeneous differential equation (17) for ϕ_2, for which the source term is described by a known function ϕ_1.

4. Stationary Solution

To obtain a stationary solution from (12) and (17), we adopt the renormalization method introduced by Kodama and Taniuti [23] to eliminate the secular behavior up to the second-order potential. According to this method, (12) can be added to (17) to yield

$$\widetilde{K}(\phi_1) + \sum_{n\geq 2}\varepsilon^n\widetilde{L}(\phi_1)\phi_n = \sum_{n\geq 2}\varepsilon^n S_n, \quad (20)$$

where S_2 represents the right-hand side of (17). Adding $\sum_{n\geq 1}\varepsilon^n\delta\Omega\frac{\partial\phi_n}{\partial\xi}$ to both sides of (20), where $\delta\Omega = \varepsilon\Omega_1 + \varepsilon^2\Omega_2 + \varepsilon^3\Omega_3 + \ldots$, with coefficients to be determined later, Ω_n are determined successively to cancel out the resonant term in S_n. Then, (12) and (17) may be written as

$$\frac{\partial\widetilde{\phi}_1}{\partial\tau} + A\widetilde{\phi}_1\frac{\partial\widetilde{\phi}_1}{\partial\xi} + \frac{1}{2}B\frac{\partial^3\widetilde{\phi}_1}{\partial\xi^3} + \delta\Omega\frac{\partial\widetilde{\phi}_1}{\partial\xi} = 0, \quad (21)$$

$$\frac{\partial\widetilde{\phi}_2}{\partial\tau}+A\frac{\partial}{\partial\xi}(\widetilde{\phi}_1\widetilde{\phi}_2)+\frac{1}{2}B\frac{\partial^3\widetilde{\phi}_2}{\partial\xi^3}+\delta\Omega\frac{\partial\widetilde{\phi}_2}{\partial\xi} = S_2(\widetilde{\phi}_1)+\Omega_1\frac{\partial\widetilde{\phi}_1}{\partial\xi}. \quad (22)$$

The parameter $\delta\Omega$ in (21) and (22) can be determined from the conditions that the resonant terms in $S_2\left(\widetilde{\phi}_1\right)$ may be replaced out by the terms $\delta\Omega\partial\widetilde{\phi}_1/\partial\xi$ in (21) [24].

Let us introduce the variable

$$\boldsymbol{\eta}=\boldsymbol{\xi}-(\Omega+\delta\Omega)\boldsymbol{\tau}, \quad (23)$$

where the parameter Ω is related to the Mach number $M=\Omega/C_{\mathrm{d}}$ by

$$\Omega+\delta\Omega\equiv M-1=\Delta M,$$

with Ω being the soliton velocity and C_{d} is the dust-acoustic velocity.

Integrating (21) and (22) with respect to the new variable η and using the appropriate vanishing boundary conditions for $\widetilde{\phi}_1(\eta)$ and $\widetilde{\phi}_2(\eta)$ and their derivatives up to second order as $|\eta|\to\infty$, we obtain

$$\frac{\mathrm{d}^2\widetilde{\phi}_1}{\mathrm{d}\eta^2}+B^{-1}(A\widetilde{\phi}_1-2\Omega)\widetilde{\phi}_1=0, \quad (24)$$

$$\frac{\mathrm{d}^2\widetilde{\phi}_2}{\mathrm{d}\eta^2}+2B^{-1}(A\widetilde{\phi}_1-\Omega)\widetilde{\phi}_2 = 2B^{-1}\int_{-\infty}^{\eta}\left[S_2(\widetilde{\phi}_1)+\Omega_1\frac{\mathrm{d}\widetilde{\phi}_1}{\mathrm{d}\eta}\right]\mathrm{d}\eta. \quad (25)$$

The one-soliton solution of (24) is given by

$$\widetilde{\phi}_1=\phi_{1\mathrm{m}}\,\mathrm{sech}^2\,(D\eta), \quad (26)$$

where the soliton amplitude $\phi_{1\mathrm{m}}$ and the soliton width D^{-1} are given by

$$\phi_{1\mathrm{m}}=\frac{3\Omega}{A},\quad D^{-1}=\sqrt{\frac{2B}{\Omega}}. \quad (27)$$

Substituting (26) in (24), the source term of (24) becomes

$$2B^{-1}\int_{-\infty}^{\eta}\left[S_2(\widetilde{\phi}_1)+\Omega_1\frac{\mathrm{d}\widetilde{\phi}_1}{\mathrm{d}\eta}\right]\mathrm{d}\eta = 2B^{-1}(\Omega_1-16D^4A_4)\phi_{1\mathrm{m}}\,\mathrm{sech}^2(D\eta) \quad (28)$$
$$+\frac{2\phi_{1\mathrm{m}}^2\Omega}{B^2}\alpha_1\,\mathrm{sech}^4\,(D\eta)+\frac{\phi_{1\mathrm{m}}^2\Omega}{B^2}\alpha_2\,\mathrm{sech}^6\,D(\eta),$$

where

$$\alpha_1=2A_2+A_3+\frac{10A}{B}A_4,$$
$$\alpha_2=\frac{2B}{A}A_1-6A_2-2A_3-\frac{20A}{B}A_4.$$

In order to cancel the resonant terms in $S(\widetilde{\phi}_1)$, the value of $\delta\Omega$ should be

$$\Omega_1=16D^4A_4. \quad (29)$$

To solve (28), we introduce the independent variable

$$\Psi=\tanh(\eta D). \quad (30)$$

Equation (28) becomes

$$\frac{\mathrm{d}}{\mathrm{d}\Psi}\left[(1-\Psi^2)\frac{\mathrm{d}\widetilde{\phi}_2}{\mathrm{d}\mu}\right]+\left[3(3+1)-\frac{2^2}{1-\Psi^2}\right]\widetilde{\phi}_2 = \alpha_3(1-\Psi^2)+\alpha_4(1-\Psi^2)^2, \quad (31)$$

where

$$\alpha_3=\frac{4\phi_{1\mathrm{m}}^2}{B}\alpha_1,\quad \alpha_4=\frac{2\phi_{1\mathrm{m}}^2}{B}\alpha_2.$$

Two independent solutions of the homogeneous part of (31) are given by the associated Legendre function of first and second kind:

$$P_3^2=15\Psi(1-\Psi^2), \quad (32)$$

$$Q_3^2=\frac{15}{2}\Psi(1-\Psi^2)\ln\left(\frac{1+\Psi}{1-\Psi}\right)+2(1-\Psi^2)^{-1}-15(1-\Psi^2)^2+5, \quad (33)$$

and the complementary solution of (31) is given by

$$\widetilde{\phi}_{2\mathrm{c}}=C_1(15\Psi(1-\Psi^2)) +C_2(\frac{15}{2}\Psi(1-\Psi^2)\ln\left(\frac{1+\Psi}{1-\Psi}\right) +2(1-\Psi^2)^{-1}-15(1-\Psi^2)+5). \quad (34)$$

By applying the method of variation of parameters, the particular solution of (31) is given by

$$\widetilde{\phi}_{2\mathrm{p}}=L_1(\Psi)P_3^2(\Psi)+L_2(\Psi)Q_3^2(\Psi), \quad (35)$$

where $L_1(\Psi)$ and $L_2(\Psi)$ are given by

$$L_1(\Psi)=-\int\frac{Q_3^2(\Psi)T(\Psi)}{(1-\Psi^2)W(P_3^2(\Psi),Q_3^2(\Psi))}\mathrm{d}\Psi,$$

$$L_2(\Psi) = \int \frac{P_3^2(\Psi)T(\Psi)}{(1-\Psi^2)W(P_3^2(\Psi),Q_3^2(\Psi))}\mathrm{d}\mu,$$

with

$$T(\mu) = \alpha_3(1-\Psi^2) + \alpha_4(1-\Psi^2)^2,$$

$$W(P_3^2,Q_3^2) = P_3^2\frac{\mathrm{d}Q_3^2}{\mathrm{d}\Psi} - Q_3^2\frac{\mathrm{d}P_3^2}{\mathrm{d}\Psi} = \frac{120}{1-\Psi^2}.$$

Using P_3^2 and Q_3^2, then $L_1(\Psi)$ and $L_2(\Psi)$ reduce to

$$L_1(\mu) = \frac{1}{32}(1-\Psi^2)^3(\frac{1}{3}\alpha_3+\frac{1}{4}\alpha_4(1-\Psi^2))\ln(\frac{1+\mu}{1-\mu}) + (\alpha_5)\Psi + (\alpha_6)\Psi^3 + (\alpha_7)\Psi^5 + \frac{1}{64}\Psi^7,$$

$$L_2(\mu) = -\frac{1}{16}(1-\Psi^2)^3(\frac{1}{3}\alpha_3 + \frac{1}{4}(1-\Psi^2)\alpha_4),$$

where

$$\alpha_5 = (-\frac{1}{48}+\frac{1}{15})\alpha_3 + (-\frac{1}{64}+\frac{1}{15})\alpha_4,$$

$$\alpha_6 = (-\frac{1}{18})\alpha_3 + (-\frac{33}{360}+\frac{1}{64})\alpha_4,$$

$$\alpha_7 = (\frac{1}{48})\alpha_3 + (-\frac{3}{320}+\frac{1}{15})\alpha_4,$$

and the particular solution is given by

$$\widetilde{\phi}_{2\mathrm{p}} = \frac{\phi_0^2 D^2}{\omega}(1-\Psi^2)[\alpha_8 - \alpha_2(1-\Psi^2)], \quad (36)$$

where

$$\alpha_8 = \frac{6BA_1}{A} - \frac{10}{3}A_2 - \frac{4}{3}A_3 - \frac{20A}{3B}A_4.$$

In (34) the first term is the secular one, which can be eliminated by renormalization of the amplitude. Also, the boundary conditions $\widetilde{\phi}_2 = 0$ as $\eta \to \infty$ produce $C_2 = 0$. Thus, only the particular solution contributes to $\widetilde{\phi}_2$.

Expressing (36) in terms of the old variable η, the solution of (31) is given by

$$\widetilde{\phi}_{2\mathrm{p}} = \frac{\phi_{1\mathrm{m}}^2 D^2}{\Omega}\mathrm{sech}^2(D\eta)[\alpha_{10} + \alpha_2\tanh^2(D\eta)]. \quad (37)$$

The stationary soliton solution for DA waves is given by

$$\widetilde{\phi}(\eta) = \widetilde{\phi}_1(\eta) + \widetilde{\phi}_2(\eta) = (\phi_{1\mathrm{m}} + \frac{\phi_0^2 D^2}{\Omega}\alpha_{11})\mathrm{sech}^2(D\eta) - \alpha_2\frac{\phi_{1\mathrm{m}}^2 D^2}{\Omega}\mathrm{sech}^4(D\eta), \quad (38)$$

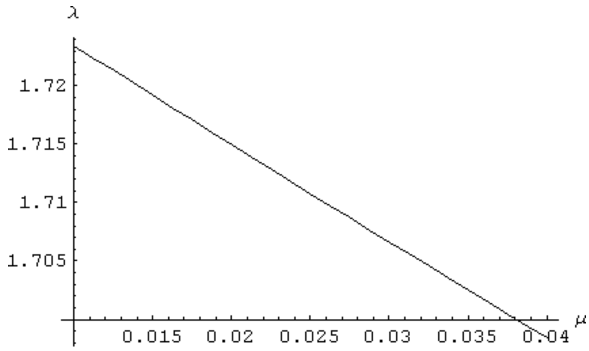

Fig. 1. Dependence of the phase velocity λ on μ.

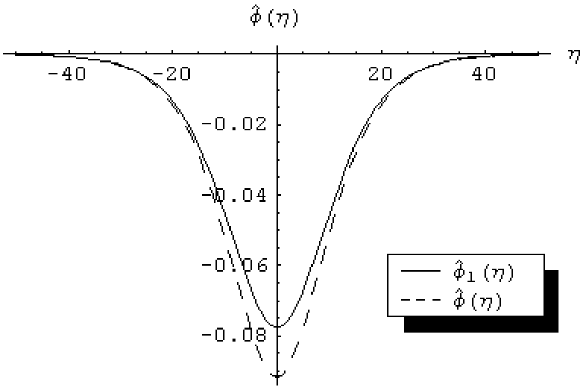

Fig. 2. Comparison between $\tilde{\phi}(\eta)$ and $\tilde{\phi}_1(\eta)$ with respect to η for $\mu = 0.01$ and $\Omega = 0.03$.

where

$$\alpha_{10} = \frac{3BA_1}{A} - \frac{1}{2}A_2 + \frac{2}{3}A_3 + \frac{10A}{3B}A_4,$$

$$\alpha_{11} = \alpha_{10} + \alpha_2,$$

$$\Omega = B^2[(1+\frac{16A_4\Delta M}{B^2})^{\frac{1}{2}} - 1]/(8A_4),$$

and the soliton width is given by

$$D^{-1} = (\frac{2B}{\triangle M})^{\frac{1}{2}}(1+\frac{2A_4\triangle M}{B^2}).$$

5. Conclusion and Remarks

In the above analysis we have investigated the effect of higher-order corrections on nonlinear dust-acoustic waves in an unmagnetized collisionless mesospheric dusty plasma consisting of positively and negatively charged dust fluids. The application of the reductive

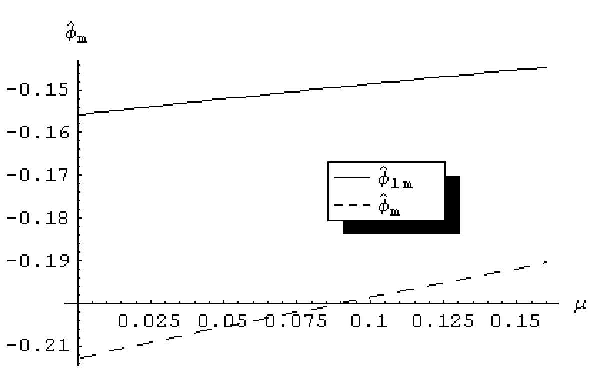

Fig. 3. Dependence of the higher-order corrections amplitude $\tilde{\phi}_m$ and first-order amplitude $\tilde{\phi}_{1m}$ on μ for $\Omega = 0.06$.

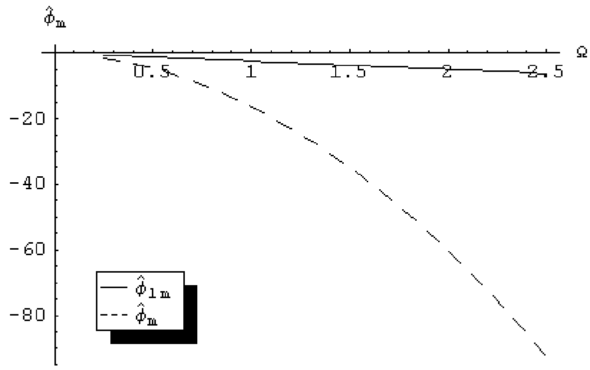

Fig. 4. Variation of the higher-order corrections amplitude $\tilde{\phi}_m$ and first-order amplitude $\tilde{\phi}_{1m}$ on Ω for $\mu = 0.1$.

perturbation theory to the basic set of fluid equations leads to a KdV equation which describes the nonlinear evolution of the dust-acoustic solitary waves. To investigate the effects of the higher-order contributions to nonlinear dust-acoustic solitary waves, we have gone to the next order in perturbation theory. A linearized KdV-type equation with an inhomogeneous term has been obtained. To obtain a stationary solution for the coupled equations (12) and (17), we have applied the renormalization method [24]. This solution includes the effects of the higher-order nonlinearity and dispersion on the nonlinear dust-acoustic solitary waves. The dependance of the phase velocity λ on μ is shown in Figure 1. In Fig. 2 it is clear that the higher-order correction increases the rarefactive amplitude of the nonlinear dust-acoustic solitary waves. In Fig. 3, the variations of the parameter μ decrease the amplitude of the solitons for the first-order potential $\tilde{\phi}_1$ and the higher-order correction $\tilde{\phi} = \tilde{\phi}_1(\eta) + \tilde{\phi}_2(\eta)$. On the other hand, Fig. 4 shows that the variation of the velocity Ω increases the higher-order correction $\tilde{\phi}(\eta)$ by a large value. In conclusion the model and the results presented here may be applicable to some dusty space plasma environments, the Earth's environment, lower and upper mesosphere.

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